Math and Art: Collaborative Practices

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Joint Mathematics Meetings, 2012

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Motivating Axiom

Definition

Interdisciplinary collaboration is the practice of multiple individuals from multiple disciplines engaging in creative acts which mutually benefit and enrich each discipline.

Axiom

It is possible, meaningful, and desirable for artists and mathematicians to engage in interdisciplinary collaboration.

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What Collaboration Looks Like

Collaboration between mathematicians and artists will

- Actively engage mathematicians and artists in a project
- Contribute to the field of Art
- Contribute to the field of Mathematics
- Inspire new directions in Art
- Inspire new directions in Mathematics

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$$\frac{AB}{PO} = 2 \Rightarrow$$
 triangles collapse

• $\frac{\overrightarrow{AB}}{\overrightarrow{PQ}} = \frac{1}{3}(2\sqrt{6}+1) \approx 1.9663265 \Rightarrow \text{structure is rigid}$

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Anaylsis of Collaboration

Artist uses own intuition to create art object

- Mathematician inspired to create mathematical model of the object and analyze it
- Communication goes "one way"
 - Art informs mathematics
 - Mathematics informs art
 - "I must confess that I don't understand the mathematics of your essay, but I do get immense satisfaction in looking at the equations and knowing that they relate directly to something that has 'popped' into my brain" - Robinson

• Math object largely illustrative of art object.

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Further Collaboration

• Coxeter communicates to Robinson:

Theorem (Soddy 1936)

There exists a sequence of spheres in \mathbb{R}^3 in which any five consecutive spheres are all mutually tangent. The radii of the spheres progress geometrically in proportion equal to the root of $x^5 - x^4 - x^3 - x^2 + x + 1 = 0$ which is between 1 and 2.

• Robinson's response: "Firmament" (1996)

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Communication goes "one way"

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Homology Project

- Use results from R. Ghrist, V. de Silva, etc., and build on these
- Basic algebraic topology (homology of simplicial complexes, some geometry)
- Coverage of sensor networks
- Locations of holes in network
- Create an interactive immersive environment
- "Record" each performance in the creation of a 3D printed object
- OpenSource project

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Custom Hardware



• Short range RF TX/RX

- Custom designed circuits based on Arduino
- Homological computations done in Sage
- 3D printing on MakerBot Thing-O-Matic

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INTD 235 Math/Art: Collaborative Practices

- Class of 13 students, various academic disciplines (10 majors represented)
- Three self-organized groups (5, 5, 3)
- Build arduino sensors from components on custom printed PC boards
- Utilize ideas from Applied Algebraic Topology
- Design and build immersive interactive environments using the sensor network as a medium

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Student Projects



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Student Projects



Bocanegra, Kronholm Math and Art: Collaborative Practices

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- Open source, therefore customizable by other artists
- Allows for hands-on exploration of mathematical objects
 - Unit disc graphs of points in \mathbb{R}^2 and \mathbb{R}^3
 - Rips complexes
 - Configurations of points in \mathbb{R}^2 and \mathbb{R}^3
- Interesting mathematical questions
 - What is the "best" way to represent the performance as a 3D object?
 - Should preserve node-node communications
 - Should preserve topology of the network
 - What other topological information can be determined about the network based on communication data?

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